Optimizing LDI Strategies

Executive summary

As plan sponsors continue their journey with Liability Driven Investing (LDI), they can better realize their funded status objectives and outcomes by refining their approach to asset allocation decisions. Rather than considering allocations for return-seeking assets and a liability-hedging portfolio independently, they can adopt a comprehensive strategy for allocating all plan assets directly alongside liabilities. For years, LGIM America has worked with clients to tailor funded status objectives to the specifics of their plan. This experience has helped us analyze optimization techniques that can help all plans manage funded status in the context of an LDI framework. Even plan sponsors that have adopted an asset liability lens must consider several key issues, including:

- How to tailor funding status objectives to their plan's goals.
- How volatility drag affects funded status returns, and how cash outflows from benefit payments exacerbate volatility drag.
- How using leverage and a more active LDI investing approach can add value to their asset allocation strategy.

Our total plan-level approach to pension portfolio decision-making can help address these considerations and generate better potential outcomes.

Introduction

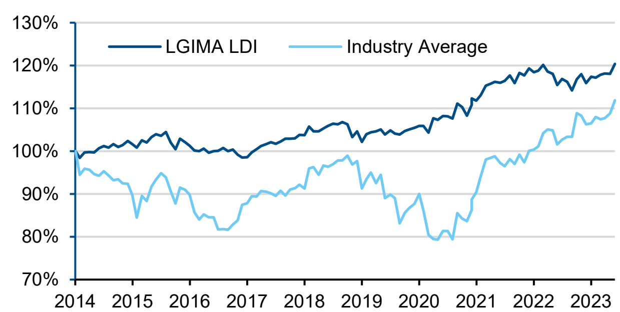
Our goal is to help pensions develop ever-improving approaches to managing funded status outcomes. Since pensions manage funding outcomes with a pool of assets against a given liability, we believe it is important to consider the return and volatility of the plan's funded status directly, rather than optimizing asset returns and volatility and adding a liability hedging component.

This paper updates and extends our Liability Driven Investing (LDI) framework by utilizing familiar optimization techniques applied to risk and return metrics better suited for investing in an asset liability framework. Specifically, we focus on surplus and funded ratio return versus surplus volatility and funded ratio volatility. We further demonstrate the importance of volatility management in the asset-liability space by illustrating how unintentional funded status volatility conspires with cash flows to generate an additional drag on achieving the desired funded status outcome. This effect is conceptually similar to the difference between arithmetic and geometric returns (and is why geometric returns are used for portfolio optimizations), and it is particularly amplified for underfunded pensions. Finally, we examine how the use of leverage and the addition of an active LDI approach can help plan sponsors make asset allocation decisions to help manage the risk they care about most—meeting their benefit obligations—while reducing required cash contributions and balance sheet volatility.

LDI's continued evolution

The widespread adoption of LDI strategies has helped plan sponsors better manage volatility in their funded status. Our first series of LDI papers outlined successive steps sponsors may take toward fully hedging their liabilities. The most recent update to that thought leadership noted that, critically, we see our clients implementing LDI as a journey where the approach is refined over time. As shown in Figure 1, these approaches clearly demonstrate reduced funded status risk for plan sponsors. Nevertheless, markets continuously challenge sponsors' efforts to prudently manage the risk of a plan failing to meet its future obligations. For example, we commonly hear from clients struggling to balance the competing priorities of minimizing cash contributions and minimizing balance sheet volatility after meeting their benefit obligations. This challenge often feels to plan sponsors like a difficult choice between seeking higher returns or increasing their liability hedge.

Figure 1: LDI vs. industry average



Source: LGIM America. For illustrative purposes only.

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We believe the approaches in this paper bring into focus the interactions between asset allocation and liability hedging decisions, alongside plan sponsor constraints. Viewed in this comprehensive way, plan sponsors can assess more clearly the costs and benefits of seeking higher returns, higher hedge ratios—or, by reconsidering leverage, both.

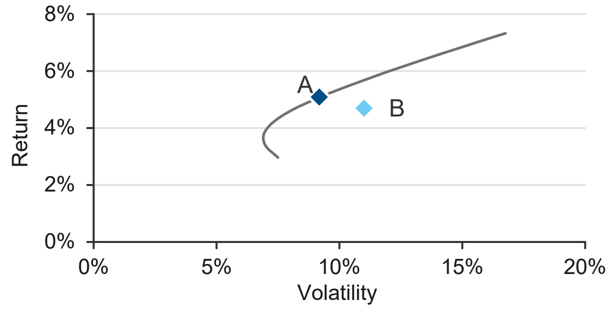
A practical mean-variance approach to asset-liability management

LGIM America has previous written articles regarding the adoption of an LDI program as an evolutionary journey—one in which a sponsor moves from a fixed income allocation with a market-based benchmark toward a longer duration benchmark and then toward a fully customized LDI portfolio matched to their specific liabilities. This evolution typically starts from a mean-variance efficient portfolio—that is, one based on maximizing total return for a given level of risk subject to certain constraints and preferences. Mean-variance optimization generates target allocations to equities, fixed income and other asset classes as appropriate. Plan sponsors then implement all or a portion of the fixed income target to better match their liabilities, either by maximizing the hedge with the capital allotted in their strategic portfolio or by targeting a hedge ratio and constructing the LDI portfolio alongside other fixed income exposures (e.g., High Yield or Emerging Market Debt).

As the plan sponsor's LDI implementation evolves, they may recategorize assets as Return-seeking Assets (RSA) and Liability-hedging Assets (LHA) in a shift toward maximizing liability hedging while retaining enough return potential to close any funding gap. Meanwhile, the plan's portfolio may remain allocated according to mean-variance optimal weights—yet in practice, the portfolio may no longer be mean-variance optimal, nor the optimal balance for managing funded status.

To illustrate this point, consider the hypothetical efficient frontier of asset returns and volatility in Figure 2. Portfolio A is a typical mean-variance optimal portfolio within a given set of assumptions and constraints and considering only a plan's investible assets. The fixed income component of Portfolio A is primarily composed of broad market exposure (e.g., Bloomberg US Aggregate). Portfolio B represents the same asset allocation weights—meaning the total fixed income and other asset class allocations are held constant—but replaces the aggregate fixed income exposure with a blend of longer duration credit and Treasuries. This blend is designed to first hedge 100% of a plan's liability interest rate exposure and then to maximize the liability credit spread hedge ratio. We see that an LDI extension of an asset-only efficient portfolio is no longer efficient.

Figure 2: Hypothetical efficient frontier of asset returns and volatility

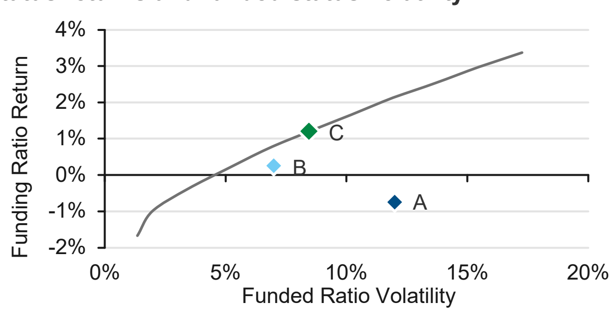


Source: LGIMA. For illustrative purposes only.

A plan sponsor's goal is not only to meet its full benefit obligations but to do so with the least possible stress to participants and the sponsor. This goal still translates naturally to an optimization framework—particularly for those familiar with the mean-variance concept described above—because the liability is a natural "short" position in the context of the total pension portfolio (see *Appendix 2* for full details). Specifically, a sponsor seeks to grow the asset portfolio faster than liabilities, but with as little deviation between the two as possible. Said differently, a sponsor will want to maximize their funded status while also minimizing funded status volatility. To meet this objective, a sponsor can choose to focus on plan surplus or funded ratio to anchor this approach (we discuss important considerations involved in this decision later in this paper).

To further illustrate our point, next consider an efficient frontier of funded status returns and funded status volatility. Portfolios A and B are the same as above. We see that Portfolio A is far from optimal on the funded status frontier. Portfolio B—the LDI extension of Portfolio A—is closer but still sub-optimal. Portfolio C is fully optimized in an asset-liability context.

Figure 3: Hypothetical efficient frontier of funded status returns and funded status volatility



Source: LGIMA. For illustrative purposes only.

Not only is the evolution to a funded status optimization framework natural, but the benefits are quite transparent when you consider the potential impact of volatility drag and cash outflows on a plan's returns.

Note: Inputs used in the efficient frontier analysis described above and in Figures 2-6 are proprietary to LGIM Ameirca, but available upon request.

Volatility drag

We believe that a funded status optimization framework benefits plan sponsors by creating a natural method for evaluating quantitative and qualitative trade-offs between sponsors' obligations and competing constraints. This approach creates value through plan governance, plan risk management and sponsor liquidity management. Further, there is a fairly simple concept that highlights the potential for this approach to lead to better outcomes for participants and plan sponsors: the importance of compound returns applied to funded status.

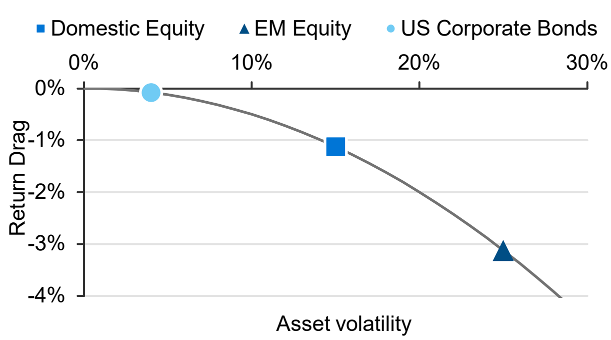
One of the basic facts of financial mathematics is that there is a difference between arithmetic and geometric returns. Arithmetic returns are used to understand growth over a fixed period, while geometric returns are used to understand growth over multiple periods. Geometric returns capture how the compounding of volatile returns impacts a portfolio's growth potential. The difference between the expected arithmetic and expected geometric returns is referred to as volatility drag. On paper, volatility drag is the simple result of a mathematical rule known as Jensen's Inequality (and a close cousin of the Triangle Inequality from high school geometry). It is a powerful force that can erode a portfolio's growth potential.

Conventional discussions of volatility drag describe it as half the portfolio variance using the following approximation:

$$\mu_{geometric} \approx \mu_{arithmetic} - \frac{\sigma^2}{2}$$

As shown in Figure 4, this formula reveals that risker assets such as equities can face a substantial headwind from volatility drag. That said, the standard formula above is only an approximation derived from making specific assumptions about a portfolio's returns, and it misses important factors such as the fact that volatility drag increases with time horizon.

Figure 4: Return impact from increasing volatility

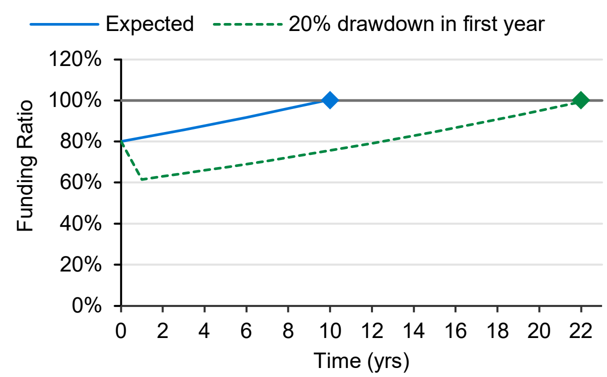


Source: LGIM America. For illustrative purposes only.

For a pension concerned with funding ratio returns or changes in funding surplus, rather than asset returns in isolation, the impact of regular cash outflows for benefit payments exacerbates the challenge of estimating volatility drag. These outflows increase the impact of poor returns, a phenomenon often referred to as sequence of return risk.

This concept is likely familiar to many readers, if not intuitively, then from experience. The figure below highlights this concept, which can have a particularly large impact on underfunded plans. We see that a volatility shock can significantly impede a plan's progress toward its funded status objective. By focusing objectives on funded status volatility, however, plans may be able to better protect against and recover from otherwise large drawdowns in return-seeking assets.

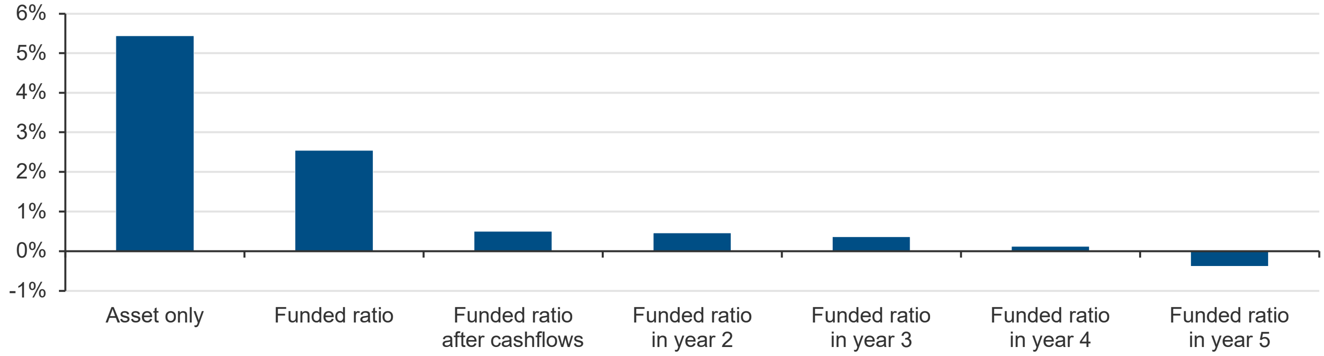
Figure 5: Drawdown impact on funding time horizon



Source: LGIM America. For illustrative purposes only.

The combination of approximating volatility drag, regular cash flows and varying time horizons can either make the volatility drag higher or lower than predicted by the standard formula. These dynamics are illustrated in Figure 6 on the following page, which shows how each of these factors can affect the expected return for a sample pension plan. Here we can see both how the funding ratio return is significantly reduced by the impact of cash flows and continues to decline even more in future years. In this example, the expected growth rate of the portfolio declines by almost 90 basis points after five years.

Figure 6: Illustrative return assumptions including the effects of cash flows and volatility



Source: LGIM America. For illustrative purposes only.

Hedging decisions for different funding objectives

Once liability-sensitive investors understand the harmful effects of the collusion between cash outflows and asset volatility on portfolio returns, they can take steps to mitigate those effects. There are three primary decisions that plan sponsors can make or re-evaluate to overcome volatility drag:

1. Targeting a funding ratio (i.e., assets divided by liabilities) or funding surplus (i.e., assets minus liabilities) and adopting a corresponding liability hedging approach
2. Updating policy allocations optimized for that target
3. Employing strategies for potential return enhancement

Funded status targets

The concept of hedging initially appears straightforward: reduce the uncertainty created by market movements. At the very least, pension plans should aim to reduce risks that are unintentional, such as the interest rate risk embedded in pension liabilities. Deciding to hedge interest rate risk is only the first step, however, as plan sponsors must then make a few more decisions—including the amount of risk to hedge and, critically, what the sponsor is trying to achieve with that hedge. The liability hedging approach and its corresponding efficacy should differ depending on whether the sponsor seeks to manage the funding ratio or the funding surplus.

If the objective is to ensure that the funding ratio remains stable regardless of changes in liability movements, then we are referring to a funding ratio hedge. On the other hand, the objective of managing a funding surplus is to ensure that the funded status remains unchanged when liabilities move. While the difference might appear subtle at first, it has some very practical implications.

Let's consider a simple plan with the following characteristics:

	Plan's position
Assets ($m)	80
Liabilities ($m)	100
Funded status: surplus/(deficit) ($m)	-20
Funding ratio	80%

Let's further assume a falling interest-rate environment, which causes liabilities to increase by $10 million. (The following illustration—underfunded plan, falling rates—is only one of several possible scenarios, and we assume all other factors are unchanged to focus on the impact of a change in rates.)

With a funded status hedge, the objective is to ensure that the assets also increase by $10 million, neutralizing the impact of rising liabilities on the deficit. In doing so, the plan's financials evolve as follows:

Falling rates environment, underfunded plan	FS hedge
Initial portfolio value ($m)	80
Change in value from hedging ($m)	10
Portfolio value after interest rates move ($m)	90
Liabilities after interest rates move ($m)	110
Funded status ($m)	**-20**
Funding ratio	**82%**

Next, let's assume that the plan sponsor wishes instead to keep the plan's funding level unchanged as a result of the change in interest rates. Hedging the funding ratio will aim to ensure that the funding level remains unchanged at 80% as liabilities increase by $10 million.

Falling rates environment, underfunded plan	FR hedge
Initial portfolio value ($m)	80
Change in value from hedging ($m)	8
Portfolio value after interest rates move ($m)	88
Liabilities after interest rates move ($m)	110
Funded status ($m)	**-22**
Funding ratio	**80%**

Plan sponsors achieve their objectives in both illustrations. However, it is important to note that hedging either the funding surplus or funding ratio came at the expense of the other. Hedging 100% of the interest risk on a funding surplus basis resulted in an increase in funding ratio from 80% to 82%, whereas a funding ratio hedge resulted in a larger deficit (from -20 to -22) even though the ratio was unchanged. If the objective is to maintain the plan's funded status, the funding ratio cannot generally be maintained. Conversely, a plan sponsor that wants to maintain a funding level will not be able to keep the plan's funded status unchanged.

To implement a funded status hedge, the portfolio manager will ensure the dollar interest rate sensitivity of the portfolio matches the dollar interest rate sensitivity of the liabilities (i.e., the DV01 should match, adjusted for the target hedge ratio). For a funding ratio hedge, the portfolio manager will set the portfolio's interest rate sensitivity equal to that of the liabilities (adjusted for the target hedge ratio) *times the funding ratio.*

FS hedge: DV01 of liabilities x hedge ratio target
FR hedge: DV01 of liabilities x hedge ratio target x FR

Another way to read this is:
FR hedge = **FS hedge** x **funding ratio**

The only time when hedging funded status and hedging funding ratio converge is when the plan is fully funded. At that point, the funding ratio is equal to 100% (funded status is nil) and hedging either one will produce the same outcome.

Going back to our example, the objective was to protect the plan against changes in interest rates. As a reminder, liabilities moved by $10 million, the funding ratio was 80% and the target interest rate hedge ratio was 100% in both cases. The portfolio manager would set the DV01 of the hedge portfolio as follows:

- Funded status hedge: $10 million x 100% = $10 million
- Funding ratio hedge: FS hedge x funding ratio = $10 million x 80% = $8 million

Based on the above illustration, it might be tempting to conclude that a funded status hedge is preferable, since the plan's overall situation is improved (deficit unchanged and improved funding level). However, the context of this example is critical: We're assuming an underfunded plan and a falling-rates environment. Let's look at what happens if the plan is still underfunded, but rates rise.

Rates rising environment, underfunded plan	FS hedge	FR hedge
Initial portfolio value ($m)	80	
Change in value from hedging ($m)	-10	-8
Portfolio value after interest rates move ($m)	70	72
Liabilities after interest rates move ($m)	90	90
Funded status ($m)	**-20**	**-18**
Funding ratio	**78%**	**80%**

This time the conclusions are reversed and hedging the funding level is preferred overall—the funding ratio remained unchanged, while the funded status improved. While we will not illustrate the scenario with an overfunded plan, it is not too difficult to extrapolate (See Appendix 1 for full illustrations). The optimal hedging approach given a plan's funded status and rate environment, holding all other factors constant, can be summarized as:

	Underfunded	Overfunded
Rates drop	**FS hedge**	**FR hedge**
Rates rise	**FR hedge**	**FS hedge**

The results are intuitive if we consider the fundamental observation that FR hedge = FS hedge x FR. If a pension plan is underfunded, hedging on a funded status basis results in the portfolio having more interest rate sensitivity than if the hedge was on a funding ratio basis. Said differently, because the funding ratio is less than 1 for an underfunded plan, FR hedge < FS hedge. The implication is that the portfolio will react more to changes in interest rates if a FS hedge is in place compared to a FR hedge for an underfunded plan.

As usual, at times a funded status hedge will be beneficial and at other times it can be less beneficial:

- As rates drop, liabilities increase and it makes sense for the hedge portfolio to have more sensitivity, which suggests a FS hedge is preferred in that scenario.
- As rates rise, however, liabilities drop and it is beneficial to have the hedge portfolio drop less (i.e. less DV01), which favors a FR hedge.

The same logic is applied for an overfunded plan and leads to the conclusions shown in the table above.

While plan sponsors know their funded status, trying to predict the direction of rates is a difficult tactical exercise. We do not advocate incorporating a directional view when setting a strategic hedging policy. Rather, our objective is to alert plan sponsors to a risk that is often overlooked, so they can put proper risk management plans in place and weigh the impact of each decision carefully.

To understand the volatility implications, we assume that in practice many plans follow a glide path that is based on funding ratio triggers, while implementing a funded status hedge. As we saw earlier, there are cases where the deficit will remain unchanged (in line with the funded status hedge) but the funding ratio could change substantially.

This is why we believe that plans looking to reduce surplus volatility should implement a funded status hedge and plan sponsors looking to mitigate funding ratio volatility should implement a funding ratio hedge. It is important to ensure the hedging metric is linked to the desired outcome. We have observed many plans creating unintended volatility because of the inconsistency between objectives and hedging implementation. Understanding whether your objective is to maintain funded status or funding ratio could be a very simple step to help reduce unintended volatility.

Optimizing asset allocation target for the funded status objective

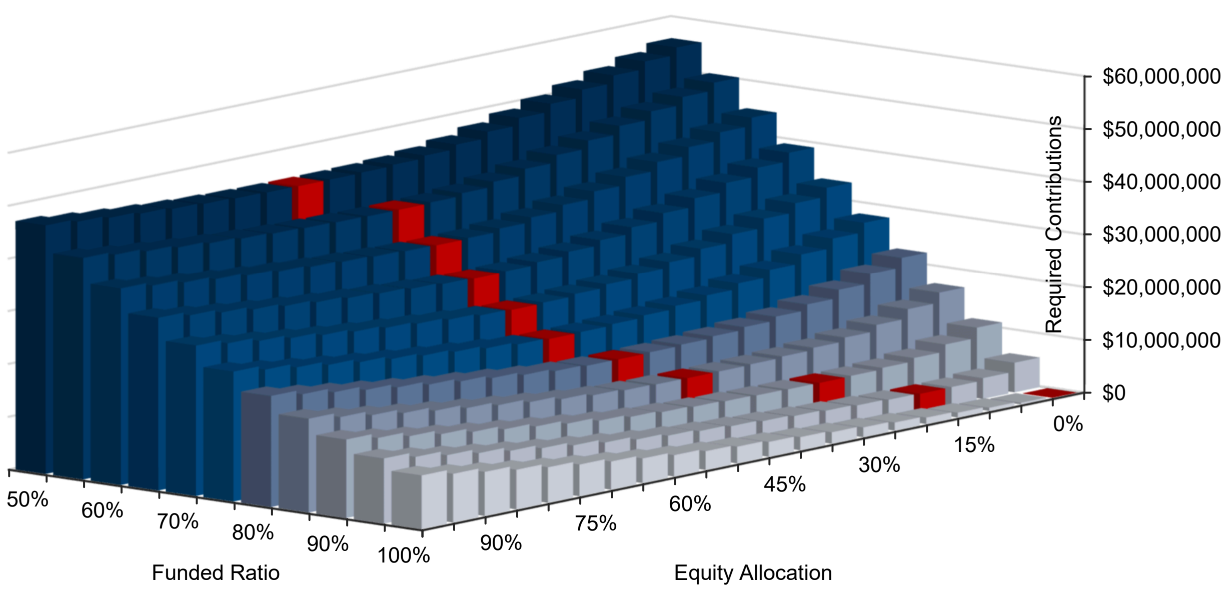
As previously noted, we believe plan sponsors using LDI strategies should optimize portfolio allocations for funded status returns and funded status volatility efficiency. However, it is difficult to approximate the potential effects of volatility drag on a pension plan using a simple, closed-form formula because of the practical considerations of cash flows and time horizon.

The good news is that a simulation-based approach is well-suited for this challenge. An additional benefit is the ability to incorporate the funded status objective and liability hedging approach into the simulation. Typically, those choices are made strategically based on considerations for both the plan and the plan sponsor. In order to remain flexible around those assumptions and needs, we focus our approach on identifying an asset allocation that minimizes future contributions from the plan sponsor to maintain the plan's funded status.

For example, Figure 7 illustrates a simple strategic asset allocation policy given only a plan's initial funding ratio. The red bars represent the equity allocation that minimizes future contributions given a funding ratio hedging approach. We see that, intuitively, the optimal equity allocation decreases as the plan becomes better funded. The relationship with volatility drag also becomes obvious. Holding a higher allocation to equities increases volatility (and volatility drag), while reducing correlation with the plan's liabilities. Because of these effects, an underfunded plan's required contributions become larger.

We also note, however, that there is a wide range of initial funded status with approximately the same optimal equity target. By no means does this imply that plan sponsors in these scenarios are forced to allocate a significant portion of assets to equity, remain exposed to the vagaries of risk markets and simply hope for the best. Plan sponsors can consider strategies to further improve outcomes. including the use of leverage and evaluating outcomes using other risk metrics.

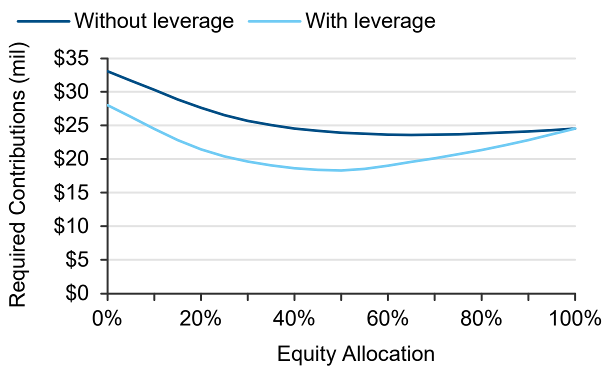
Figure 7: Optimal equity allocation to minimize future contributions



Source: LGIM America. For illustrative purposes only. This is not an investment recommendation and does not represent any particular client account, as a strategic asset allocation policy is custom-built based on the relevant client's needs and may materially differ from the illustration above.

In Figure 8, we see that utilizing leverage (e.g., by using more physical capital for the liability hedge and sourcing exposure to return-seeking assets with derivatives, a common approach for many of our clients) reduces required contributions for nearly all scenarios. The reason is that adding leverage allows plan sponsors to reduce the amount of equity they must hold and leaves more funds available to meet their hedging objectives. This combination of factors mitigates the dual effects of increased volatility drag and reduced amount of hedging that leads to higher required contributions.

Figure 8: Utilizing leverage to reduce future contributions



Source: LGIM America. For illustrative purposes only.

Finally, plan sponsors can evaluate the optimal return-seeking allocation jointly for required contributions and other risk metrics that emphasize change in funded status, risk-adjusted returns and/or tail risk. Figure 9 below shows what we believe is the optimal return-seeking allocation for a hypothetical plan when considering any of these metrics, along with a summary of the optimal range across the various approaches. For example, the optimal return-seeking allocation for a plan that is 50% funded is 60%, assuming the main objective is to minimize required contributions. If, however, the objective is to maximize funded ratio return, then the optimal return seeking allocation increases to 85%.

Performing an analysis like this allows plan sponsors to consider their funded status objective along with any particular sensitivities or constraints in a given environment (such as a large equity market drawdown). While the range of allocations might appear large, it reflects the impact of varying objectives on the plan's optimal allocation. What's more, it also provides a benchmark for a reasonable range of equity allocations within an asset-liability framework—which, in the case of well-funded plans, may be lower than plan sponsors were used to holding when they made asset-only allocation decisions. In practice, return-seeking allocation ranges will vary with plan characteristics, and plan sponsors must make practical considerations when selecting the final allocation. This framework can be adapted to include additional constraints/objectives from plan sponsors.

Return-enhancing strategies

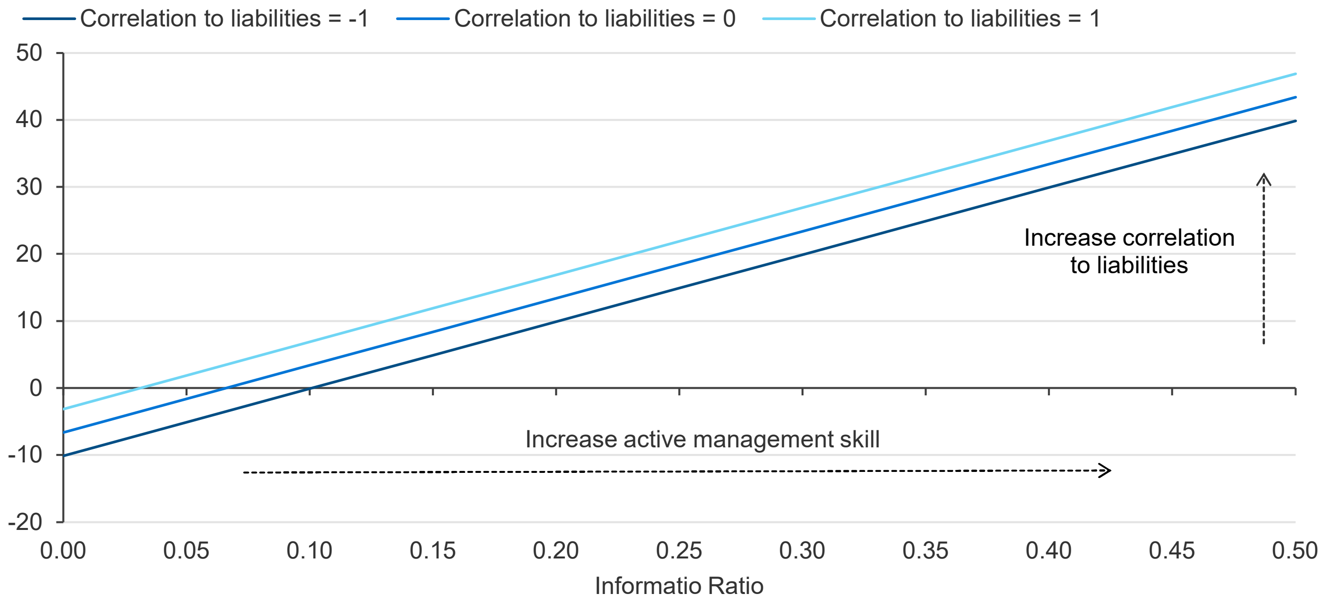
Establishing a funded status objective, hedging approach and strategic asset allocation are the most powerful ways to optimize an LDI strategy. But along with those steps, plan sponsors may consider adding value by adopting more active liability-driven investing approach. The objective of an active risk budget would be to offset the anticipated volatility drag—or perhaps even enhance funded status returns. There are several approaches to return enhancement within an active LDI program, depending on the plan sponsor's risk aversion (such as avoidance of possible additional contributions), capital availability, allocation flexibility and capacity to use derivatives and/or deploy leverage.

Figure 9: Optimal equity allocation for various objectives

	Plan's primary objective				
Funded Ratio	Funded Ratio Return	Required Contributions	Tail Weighted Return	Sharpe Ratio	Range
50%	85%	60%	85%	95%	60% - 95%
55%	60%	50%	60%	75%	50% - 75%
60%	20%	50%	20%	60%	20% - 60%
65%	25%	50%	15%	60%	15% - 60%
70%	25%	50%	15%	50%	15% - 50%
75%	40%	50%	15%	50%	15% - 50%
80%	40%	45%	20%	50%	20% - 50%
85%	50%	40%	20%	40%	20% - 50%
90%	55%	25%	20%	45%	20% - 55%
95%	60%	15%	15%	40%	15% - 60%
100%	65%	0%	15%	5%	0% - 65%

Source: LGIMA. For illustrative purposes only. This is not an investment recommendation and does not represent any particular client account, as a strategic asset allocation policy is custom-built based on the relevant client's needs and may materially differ from the illustration above.

Figure 10: Contribution to funded status return (bps)



Source: LGIMA. For illustrative purposes only. "Active portfolios" refers to any portfolio that deviates from policy allocations, and "Strategic portfolios" refers to any portfolio of long-term, static allocation targets. Table above illustrates correlation across portfolio types; it does not represent existing portfolios. Assumptions underlying this analysis are available upon request.

The chart above details the hurdle for improving funded status outcomes with an active LDI approach. We hold an active risk budget constant and see that active returns with low or negative correlation to liabilities improve outcomes, as does increasing skill as measured by information ratio[1]. For example, an "active" portfolio with zero skill (i.e., information ratio of zero) still improves outcomes as the correlation becomes more negative with liabilities, thanks to the hedging more of the liability risk. An increasing information ratio means that the plan sponsor and/or investment manager is able to more efficiently deploy the active risk budget.

The crucial observation, though, is that the active manager must be more skilled if their active views are less correlated to liabilities. This higher skill threshold is the result of the potential volatility drag that an active LDI approach (the portfolio of active views) might have on funded status outcomes.

Importantly, standard portfolio risk metrics can be adapted to an asset-liability context. In this approach we think of the liability as a short asset. From there, the extension of portfolio risk calculations is more straightforward, as we demonstrate in Appendix 2. An active risk budget can be set using a variety of metrics or constraints, including value-at-risk (VaR) or conditional value at risk (CVaR) relative to the plan's funded status. We can also understand how risk is allocated across strategic assets, tactical views and liabilities, and each position's marginal contribution to risk. From a practical perspective, the plan sponsor can then set an active risk budget based on funding ratio volatility, funding surplus volatility and/or maximum marginal cash contributions. Finally, a reasonable assumption or target for the active management information ratio provides a performance objective, funded status improvement goal and estimation of how far contributions may ultimately be reduced.

The implementation of return enhancing strategies within an LDI program may take many forms, from the simple to the more complex. For example, LDI managers may start by using flexibility around cash assets and leverage allowances to add value through optimizing funding costs or benefits that can be available to investors using derivatives (i.e., a cash "underlay"). This approach would be an example of the lowest risk and, correspondingly, lowest expected return. From there, active views on credit spread returns versus returns to duration could be applied to beat liability returns that have fixed rate and spread risks. We believe, however, that a plan sponsor managing risk holistically relative to a funded status objective is best served through a multi-asset approach to active LDI. Cross-asset views provide more opportunity to add value (i.e., increases the breadth associated the information ratio), and, more importantly, use all of the available risk measures and tools at the manager's disposal as

1 Information ratio is the excess return from active management divided by the volatility of those active returns.

described above. We believe a holistic view is critical because nearly every plan sponsor allocates to other return-seeking assets, which in turn will have varying correlations to their liabilities. Without a comprehensive view of these risks, it is not possible to fully understand how active decisions may ultimately affect funded status (rather than what asset return the decision might generate).

Conclusion

Thanks to the adoption of LDI, plan sponsors have become much more sophisticated in their approach to managing pension liabilities in the last ten years. This increased sophistication sharpens plan sponsors' and investment managers' focus on funded status outcomes. LGIM America has developed a natural extension to portfolio optimization that is based on fundamental tenets of portfolio management, but that also reflects the practicalities that pensions face.

This optimization framework enables plan sponsors to more effectively establish:

- Funded status objective
- Corresponding liability-hedging approach
- Strategic asset allocation
- Potential active risk budget to overcome inherent frictions in asset-liability management and minimize further contributions

Further, the extension of standard portfolio risk measurement into an asset-liability context provides much better insight into pension risk management and the potential effects of allocation decisions. Taken together, we believe these are crucial observations to help plan sponsors meet their benefit obligations while minimizing funded status volatility and stress on the plan sponsor.

Appendix 1

Underfunded Plan

	Falling interest rates		Rising interest rates	
	FS hedge	**FR hedge**	**FS hedge**	**FR hedge**
Initial portfolio value	80			
Initial liability value	100			
Change in liability value due to rates	10		-10	
Liabilities after interest rates move	110		90	
Change in portfolio value from hedging	10	8	-10	-8
Portfolio value after interest rates move	90	88	70	72
Funded status	-20	-22	-20	-18
Funding ratio	82%	80%	78%	80%

Overfunded Plan

	Falling interest rates		Rising interest rates	
	FS hedge	**FR hedge**	**FS hedge**	**FR hedge**
Initial portfolio value	120			
Initial liability value	100			
Change in liability value due to rates	10		-10	
Liabilities after interest rates move	110		90	
Change in portfolio value from hedging	10	12	-10	-12
Portfolio value after interest rates move	130	132	110	108
Funded status	20	22	20	18
Funding ratio	118%	120%	122%	120%

Charts depicted above are intended for illustrative purposes only.

Appendix 2:
Closed-Form Approximation of Funded Status Volatility

Let...

A_t *be the asset level at time* t

L_t *be the present value of the liability level at* t

FS_t *be the funded status ratio at* t, *i.e.*, A_t/L_t

$R_{A,t}$ *be the asset return for period ending* t

$R_{L,t}$ *be the liability return for period ending* t

$R_{FS,t}$ *be the funded status return for period ending* t

$\underset{w_{t-1}}{\longrightarrow} = \underset{w}{\rightarrow}$ *be the* $n \times 1$ *vector of portfolio weights* w_i *for assets* $i = 1$ *to* n *at time* $t - 1$

$R_{i,t}$ *be the return on asset* i *for period ending* t

Σ *be the* $n \times n$ *covariance matrix of the asset returns, annualized appropriately*

$\underset{c}{\rightarrow}$ *be the* $n \times 1$ *vector of the covariance between each asset* 1 *to* n *and the liability*

Ignore benefit payments and contributions, and assume a filtration till time t = *t-1*.

Then...

$$A_t = A_{t-1}(1 + R_{A,t}) \quad (1)$$

$$L_t = L_{t-1}(1 + R_{L,t}) \quad (2)$$

$$FS_t = \frac{A_t}{L_t} = \frac{A_{t-1}(1 + R_{A,t})}{L_{t-1}(1 + R_{L,t})} \quad (3)$$

$$R_{FS,t} = \frac{FS_t}{FS_{t-1}} - 1 \quad (4)$$

$$R_{A,t} = \sum_{i=1}^{n} R_{i,t} w_i \quad (5)$$

$$Var(R_{A,t}) = \underset{w}{\rightarrow}^T \, \Sigma \, \underset{w}{\rightarrow} \quad (6)$$

...and we are looking to find...

I. Funded status volatility in percent, $\sqrt{Var(R_{FS,t})}$

DERIVATION

Using (4) and (3)…

$$R_{FS,t} = \frac{FS_t}{FS_{t-1}} - 1 = \frac{\frac{A_t}{L_t}}{\frac{A_{t-1}}{L_{t-1}}} - 1$$

$$= \frac{\frac{A_{t-1}(1+R_{A,t})}{L_{t-1}(1+R_{L,t})}}{\frac{A_{t-1}}{L_{t-1}}} - 1$$

$$= \frac{1+R_{A,t}}{1+R_{L,t}} - 1$$

…we have…

$$Var(R_{FS,t}) = Var\left(\frac{1+R_{A,t}}{1+R_{L,t}} - 1\right)$$

Using the approximation that…

$$\frac{1+\mathrm{x}}{1+\mathrm{y}} - 1 \approx x - y \ for \ small \ x, y$$

…We then have…

$$Var(R_{FS,t}) \approx Var(R_{A,t} - R_{L,t}) = Var(R_{A,t}) + Var(R_{L,t}) - 2Cov(R_{A,t}, R_{L,t})$$

$$= \underset{w}{\rightarrow}^T \Sigma \underset{w}{\rightarrow} + \sigma^2_{R_{L,t}} - 2Cov(R_{A,t}, R_{L,t}) \qquad (7)$$

Now…

$$Cov(R_{A,t}, R_{L,t}) = Cov\left(\sum_{i=1}^{n} R_{i,t}w_i, R_{L,t}\right) \qquad (8)$$

Using the identity that…

$$Cov(aX + bY, cZ) = acCov(X, Z) + bcCov(Y, Z)$$

…With $c = 1$, we can rewrite (8) as…

$$Cov(R_{A,t}, R_{L,t}) = Cov\left(\sum_{i=1}^{n} R_{i,t}w_i, R_{L,t}\right) = Cov(R_{1,t}w_1 + R_{2,t}w_2 + \cdots + R_{n,t}w_n, R_{L,t})$$

$$= w_1Cov(R_{1,t}, R_{L,t}) + w_2Cov(R_{2,t}, R_{L,t}) + \cdots + w_nCov(R_{n,t}, R_{L,t})$$

$$= \underset{w}{\rightarrow}^T \underset{\mathrm{C}}{\rightarrow}$$

We finally rewrite (7) as…

$$Var\left(R_{FS,t}\right) = \underset{w}{\rightarrow}^{T}\ \Sigma\ \underset{w}{\rightarrow} + \sigma_{R_{L,t}}^{2} - 2\,\underset{w}{\rightarrow}^{T}\ \underset{C}{\rightarrow} \qquad (9)$$

…Yielding…

$$\textbf{\textit{Funded Status Volatility}} = \sqrt{\underset{w}{\rightarrow}^{T}\,\boldsymbol{\Sigma}\,\underset{w}{\rightarrow} + \boldsymbol{\sigma}_{R_{L,t}}^{2} - 2\,\underset{w}{\rightarrow}^{T}\,\underset{C}{\rightarrow}}$$

II. A liability is a short position in an asset

Readers may have already noted that in (7), we defined $Var\left(R_{FS,t}\right) \approx Var\left(R_{A,t} - R_{L,t}\right)$, and the R.H.S. term resembles the variance of a long-short portfolio, with a weight of +1 on the asset portfolio and a weight of -1 on the liability. That said, here we demonstrate more explicitly that a liability is equivalent to a short position in an asset, with its portfolio weight equal to -1.

In addition to the original definitions, let…

$\underset{w}{\rightarrow}^{*}$ *be the* $(n+1) \times 1$ *vector of portfolio weights* w_i *for assets* $i = 1$ *to* $n+1$, *with the* $(n+1)^{th}$ *"asset" representing the liability with* $w_{n+1} = -1$

Σ^{*} *be the* $(n+1) \times (n+1)$ *covariance matrix of the asset returns, incorporating the liability as the* $(n+1)^{th}$ asset, *annualized appropriately*

$\sigma_{i,j}$ *be the element in row i, column j of* Σ, *denoting the covairance between asset i and j (or the variance when i = j), for asset* $i, j = 1$ *to* $n+1$

Then the return of this long-short portfolio is given by…

$$R_{A-L,t} = \sum_{i=1}^{n+1} R_{i,t} w_i \qquad (10)$$

…and the variance of this portfolio is…

$$Var\left(R_{A-L,t}\right) = \underset{w}{\rightarrow}^{*T}\ \Sigma^{*}\ \underset{w}{\rightarrow}^{*} \qquad (11)$$

$$\underset{w}{\rightarrow}^{*T}\ \Sigma^{*}\ \underset{w}{\rightarrow}^{*} = \begin{pmatrix} w_1 & w_2 & \cdots & w_n & w_{n+1} \end{pmatrix} \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,n} & \sigma_{1,n+1} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,n} & \sigma_{2,n+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{n,1} & \sigma_{n,2} & \cdots & \sigma_{n,n} & \sigma_{n,n+1} \\ \sigma_{n+1,1} & \sigma_{n+1,2} & \cdots & \sigma_{n+1,n} & \sigma_{n+1,n+1} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ w_{n+1} \end{pmatrix}$$

$$= \begin{pmatrix} \underset{w}{\rightarrow}^{T} & w_{n+1} \end{pmatrix} \begin{pmatrix} \Sigma & \underset{C}{\rightarrow} \\ \underset{C}{\rightarrow}^{T} & \sigma_{n+1,n+1} \end{pmatrix} \begin{pmatrix} \underset{w}{\rightarrow} \\ w_{n+1} \end{pmatrix}$$

$$= \begin{pmatrix} \underset{w}{\rightarrow}^{T}\ \Sigma + w_{n+1}\,\underset{C}{\rightarrow}^{T} & \underset{w}{\rightarrow}^{T}\,\underset{C}{\rightarrow} + w_{n+1}\sigma_{n+1,n+1} \end{pmatrix} \begin{pmatrix} \underset{w}{\rightarrow} \\ w_{n+1} \end{pmatrix}$$

$$= \left(\underset{w}{\rightarrow}^{T}\ \Sigma\ \underset{w}{\rightarrow} + w_{n+1}\,\underset{C}{\rightarrow}^{T}\,\underset{w}{\rightarrow} + w_{n+1}\left(\underset{w}{\rightarrow}^{T}\,\underset{C}{\rightarrow} + w_{n+1}\sigma_{n+1,n+1}\right)\right) \begin{pmatrix} \underset{w}{\rightarrow} \\ w_{n+1} \end{pmatrix}$$

Given $\underset{c}{\rightarrow}^T \underset{w}{\rightarrow} = \underset{w}{\rightarrow}^T \underset{c}{\rightarrow}$, we get…

$$Var\left(R_{A-L,t}\right) = \underset{w}{\rightarrow}^T\ \Sigma\ \underset{w}{\rightarrow} + 2w_{n+1}\underset{w}{\rightarrow}^T\underset{c}{\rightarrow} + {w_{n+1}}^2\sigma_{n+1,n+1}$$

With $w_{n+1} = -1$, we then get…

$$Var\left(R_{A-L,t}\right) = \underset{w}{\rightarrow}^T\ \Sigma\ \underset{w}{\rightarrow} - 2\underset{w}{\rightarrow}^T\underset{c}{\rightarrow} + \sigma_{n+1,n+1}$$

…which, given that $\sigma_{n+1,n+1} = \sigma^2_{R_{L,t}}$, is identical to the funded status volatility formula we previously derived in (9).

We thus find that a liability is equivalent to a short position in an asset, with a portfolio weight of -1. This is a very useful result as it allows us to extend asset space portfolio mathematics to funded status space, developing several risk metrics for the latter. Before that, we present two more related measures of funded status volatility: (i) funded status volatility in funded status points and (ii.) funded status dollar surplus/shortfall volatility.

III. Funded status volatility in funded status points, $\sqrt{\boldsymbol{Var(FS_t - FS_{t-1})}}$

DERIVATION[2]

$$Var(FS_t - FS_{t-1}) = Var\left(\frac{A_t}{L_t} - \frac{A_{t-1}}{L_{t-1}}\right) = Var\left(\frac{A_{t-1}(1+R_{A,t})}{L_{t-1}(1+R_{L,t})} - \frac{A_{t-1}}{L_{t-1}}\right)$$

$$= Var\left(\frac{A_{t-1}(1+R_{A,t})}{L_{t-1}(1+R_{L,t})} - \frac{A_{t-1}}{L_{t-1}}\right)$$

$$= Var\left(\frac{A_{t-1}}{L_{t-1}}\left(\frac{1+R_{A,t}}{1+R_{L,t}} - 1\right)\right)$$

$$= \left(\frac{A_{t-1}}{L_{t-1}}\right)^2 Var\left(\frac{1+R_{A,t}}{1+R_{L,t}} - 1\right)$$

$$= \left(\frac{A_{t-1}}{L_{t-1}}\right)^2 Var\left(\frac{1+R_{A,t}}{1+R_{L,t}} - 1\right)$$

Note that we have previously already derived a formula for $Var\left(\frac{1+R_{A,t}}{1+R_{L,t}} - 1\right)$, which is simply our previous result, $Var\left(R_{FS,t}\right)$

$$Var(FS_t - FS_{t-1}) = \left(\frac{A_{t-1}}{L_{t-1}}\right)^2 Var\left(R_{FS,t}\right)$$

And therefore…

$$\sqrt{Var(FS_t - FS_{t-1})} = \sqrt{\left(\frac{A_{t-1}}{L_{t-1}}\right)^2 Var\left(R_{FS,t}\right)} = \left(\frac{A_{t-1}}{L_{t-1}}\right)\sqrt{Var(R_{FS,t})}$$

…which is simply the funded status volatility we previously derived, scaled by current funded status ratio. Unlike the former measure, funded status volatility in terms of funded status points is *not* invariant to funded status.

2 While we omit the conditional operators for simplicity, recall that we assume a filtration till time *t-1*.

IV. Funded status dollar surplus/shortfall volatility, $\sqrt{Var((A_t - L_t) - (A_{t-1} - L_{t-1}))}$

DERIVATION[3]

$$Var((A_t - L_t) - (A_{t-1} - L_{t-1})) = Var(A_t - L_t)$$

$$= Var(A_{t-1}(1 + R_{A,t}) - L_{t-1}(1 + R_{L,t}))$$

$$= Var(A_{t-1}(1 + R_{A,t})) + Var(-L_{t-1}(1 + R_{L,t})) + 2Cov(A_{t-1}(1 + R_{A,t}), -L_{t-1}(1 + R_{L,t}))$$

Using the identity that…

$$Cov(aX + k, bY + l) = abCov(X, Y)$$

With $a = k = A_{t-1}$ and $b = l = -L_{t-1}$, we get…

$$Var((A_t - L_t) - (A_{t-1} - L_{t-1})) = {A_{t-1}}^2 Var(1 + R_{A,t}) + {L_{t-1}}^2 Var(1 + R_{L,t}) - 2A_{t-1}L_{t-1}Cov(1 + R_{A,t}, 1 + R_{L,t})$$

$$= {A_{t-1}}^2 Var(R_{A,t}) + {L_{t-1}}^2 Var(R_{L,t}) - 2A_{t-1}L_{t-1}Cov(R_{A,t}, R_{L,t})$$

$$= {A_{t-1}}^2 \underset{w}{\rightarrow}^T \Sigma \underset{w}{\rightarrow} + {L_{t-1}}^2 \sigma^2_{R_{L,t}} - 2A_{t-1}L_{t-1} \underset{w}{\rightarrow}^T \underset{c}{\rightarrow}$$

Note that unlike our previous result, which was a scaled variant of our original funded status volatility metric, this result does *not* allow any such mapping.

3 Again, while we omit the conditional operators for simplicity, recall that we assume a filtration till time *t-1*.